A step function is a discontinuous function (you must lift your pencil off the paper to draw it) that, when graphed, appears as a series of disconnected line segments resembling steps on a staircase.

Two common step functions are called the **floor** and **ceiling** functions.

The floor and ceiling functions give you the **nearest integer** up or down.

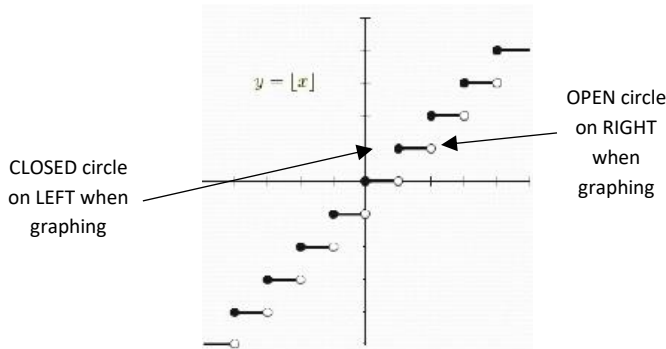Example: What is the floor and ceiling of 2.31?

The Floor of 2.31 is **2**   The Ceiling of 2.31 is **3**
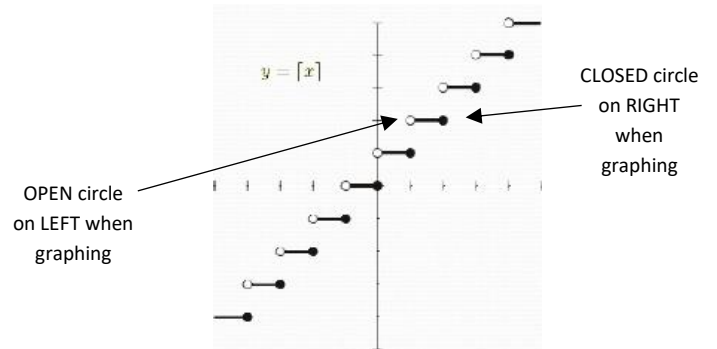
| Floor Functions (round down)	Ceiling Functions (round up)
[Graphing Calc: MATH → NUM #5: int( ]	
• bracket symbols ⌊ ⌋ with pieces only at the bottom	• bracket symbols ⌈ ⌉ with pieces only at the top
• This represents the greatest integer less than or equal to (≤) the value inside the brackets	• This represents the least integer greater than or equal to (≥) the value inside the brackets
	•

Examples of $\lfloor x \rfloor$ (ROUND DOWN to the nearest integer)

1.	$\lfloor 5 \rfloor$	=	**5**	2.	$\lfloor 3.9 \rfloor$	=	**3**
3.	$\lfloor 3.1 \rfloor$	=	**3**	4.	$\lfloor 0.99 \rfloor$	=	
5.	$\lfloor -3.2 \rfloor$	=		6.	$\lfloor -0.5 \rfloor$	=	
7.	$\lfloor -2.1 \rfloor$	=		8.	$\lfloor 6.8 \rfloor$	=	

Examples of $\lceil x \rceil$ (ROUND UP to the nearest integer)

1.	$\lceil 1.4 \rceil$	=	**2**	2.	$\lceil 4.1 \rceil$	=	**5**
3.	$\lceil 6 \rceil$	=	**6**	4.	$\lceil 0.32 \rceil$	=	
5.	$\lceil -1.8 \rceil$	=		6.	$\lceil -3.4 \rceil$	=	
7.	$\lceil -2.1 \rceil$	=		8.	$\lceil 6.8 \rceil$	=	

Picturing a number line might be helpful especially when dealing with negative #s.
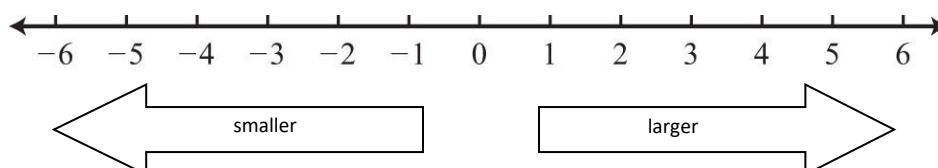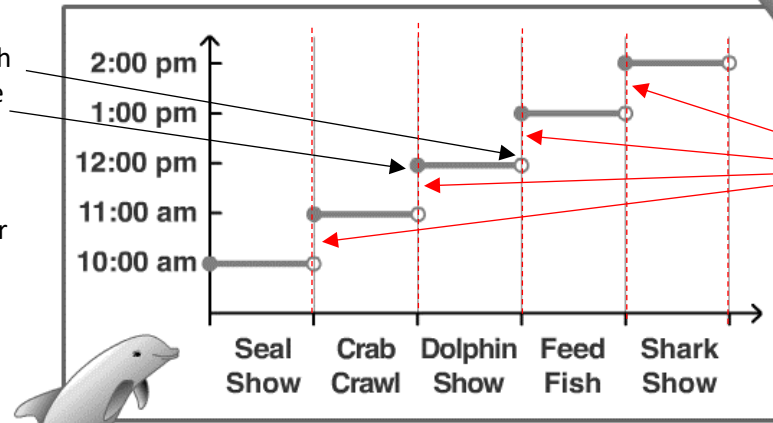
Below are examples of a STEP FUNCTION GRAPHS:

Oceanarium ... Activity Times

Steps start and stop with an open or closed circle

Values stay the same for a period of time "y = a #" equations

Intervals **DO NOT** overlap but new steps begin directly above the end of the previous step

Remember: To be a function, each x-value MUST have a **UNIQUE** y-value

Seal Show	→	10:00 ≤ t < 11:00
Crab Show	→	11:00 ≤ t < 12:00
Dolphin Show	→	12:00 ≤ t < 1:00
Feed Fish	→	1:00 ≤ t < 2:00
Shark Show	→	2:00 ≤ t < 3:00

The Cost of Postage for a Letter

Looks like it hits 2 points, but one of the points is open!
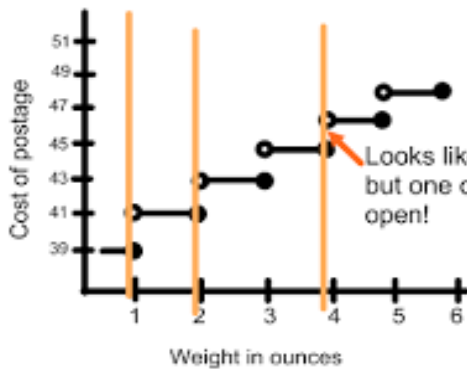
x = weight in ounces
f(x) = cost of postage

$$f(x) = \begin{cases} 0.39 & \text{if } 0 < x \le 1 \\ 0.41 & \text{if } 1 < x \le 2 \\ 0.43 & \text{if } 2 < x \le 3 \\ 0.45 & \text{if } 3 < x \le 4 \\ 0.47 & \text{if } 4 < x \le 5 \\ 0.49 & \text{if } 5 < x \le 6 \end{cases}$$

A wholesale t-shirt manufacturer charges the following prices for t-shirt orders:

- $20 per shirt for shirt orders up to 20 shirts.
- $15 per shirt for shirt between 21 and 40 shirts.
- $10 per shirt for shirt orders between 41 and 80 shirts.
- $5 per shirt for shirt orders over 80 shirts.

Cost of T-shirts
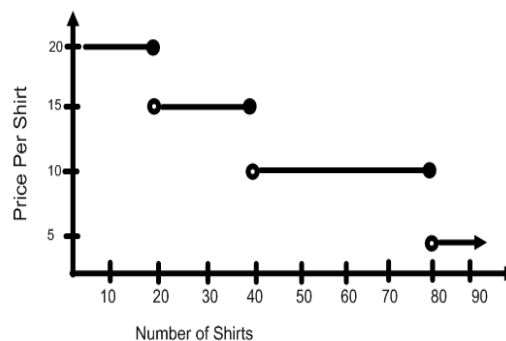
s = number of shirts
c(s) = price per shirt

$$c(s) = \begin{cases} 20 & \text{if } 0 < s \le 20 \\ 15 & \text{if } 20 < s \le 40 \\ 10 & \text{if } 40 < s \le 80 \\ 5 & \text{if } s > 80 \end{cases}$$

Name: Date: Period:

Evaluate the following Floor/Ceiling Functions.

1. $\lfloor \pi \rfloor$

2. $\lfloor -\tfrac{1}{4} \rfloor$

3. $\lceil \tfrac{1}{2} \rceil$

4. $\lceil \pi \rceil$

5. $\lfloor 1.03 \rfloor$

6. $\lfloor \tfrac{1}{2} \rfloor$

7. $\lceil 1.03 \rceil$

8. $\lceil -\tfrac{1}{4} \rceil$

9. $\lfloor -256 \rfloor$

10. $\lceil -1.5 \rceil$

11. $\lfloor -3.2 \rfloor$

12. $\lceil 2.23 \rceil$

13. $\lfloor 3.506 \rfloor$

14. $\lceil 7.29 \rceil$

15. $\lceil 2.564 \rceil$

16. $\lfloor \sqrt{5} \rfloor$

17. $\lfloor -0.7 \rfloor$

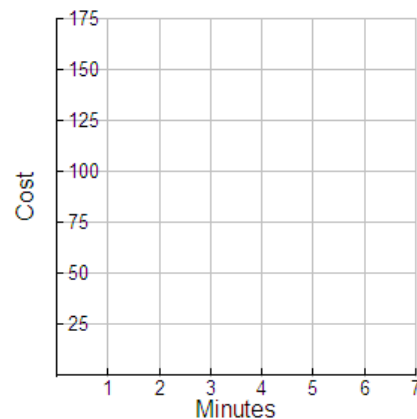18. $\lceil 8 \rceil$

19. $\lfloor \sqrt{2} \rfloor$

20. $\lceil -8.5 \rceil$

Complete the tables and graphs for the following problems.

21. Phone companies determine the price of a call by rounding the length of the call to a certain time period (usually the nearest minute).

Complete the table and graph if Bell Atlantic charges $0.25 each minute, in addition to a $0.25 connection fee for each call.

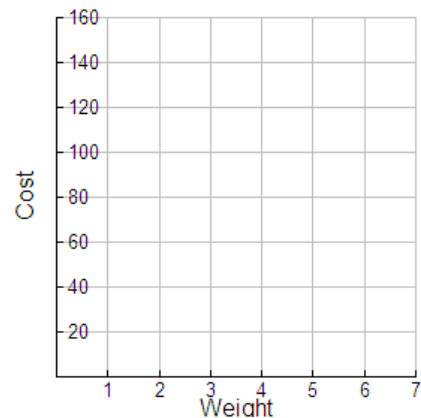| Minutes	Cost ($)
$0 < x \le 1$	
$1 < x \le 2$	
$2 < x \le 3$	
$3 < x \le 4$	
$4 < x \le 5$	
$5 < x \le 6$	

*Use cost in cents to graph.
For example, 0.20 = 20 on y-axis.

22. The table shows the cost of mailing a letter that weighs x ounces. Graph the step function and complete the second table for packages with the provided weights.

Weight (oz.)	Cost ($)	Weight (oz.)	Cost ($)
$0 < x \le 1$	0.40	1.26	0.40
$1 < x \le 2$	0.60	4.29	0.60
$2 < x \le 3$	0.90	0.98	
$3 < x \le 4$	1.10	2.55	
$4 < x \le 5$	1.30	6	
$5 < x \le 6$	1.60	3.01	

*Use cost in cents to graph.
For example, 0.20 = 20 on y-axis.

Step Function Notes